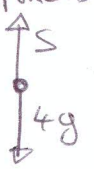
Eg (5) : (1989)

6. Define Simple Harmonic Motion.
A mass of 4 kg suspended by a light spiral spring extends it 8 cm when in equilibrium. A second mass of 2 kg is attached to the first without moving it and the combined mass is then released from rest.
(i) Prove that the motion is simple harmonic.
(ii) Find the periodic time of the ensuing motion.
(iii) Find the maximum velocity of the resulting motion.

4kg mass : $x_{equil} \Rightarrow a = 0$

Forces. Hooke $|S| = \kappa(\text{Ext})$



$|S| = \kappa(.08)$

$\therefore$ N II $\Rightarrow -|S| + mg = 0$

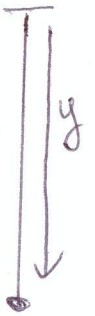
$\Rightarrow -\kappa(.08) + 4(9.8) = 0$

$\Rightarrow \boxed{\kappa = 490}$

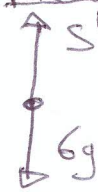
fixed for spring.

2kg added $\Rightarrow$ Mass = 6kg.

First find Equil position, y, below ceiling where $\boxed{a = 0}$



Forces



Hooke

$-S' = 490(y - l_0)$

N II $\Rightarrow -|S'| + 6g = 0$

$\Rightarrow |S'| = 6g$

$\Rightarrow 490(y - l_0) = 588$

$\Rightarrow y - l_0 = .12$

$\Rightarrow \underline{y = l_0 + .12}$

Position of equilibrium is $l_0 + .12$ below ceiling.

NEXT Examine forces at typical position $(l_0 + .12) + x$ below ceiling:

Forces

Hooke: $S = \kappa(\text{Extension})$

$|S''| = 490(.12 + x)$

N II $\Rightarrow \quad \Sigma F = ma$

$\Rightarrow -|S''| + 6g = 6a$

$\Rightarrow -490(.12 + x) + 6(9.8) = 6a$

$\Rightarrow -58.8 - 490x + 58.8 = 6a$

$\Rightarrow \underline{-81.\dot{6}x = a}$

SHM about point $l_0 + .12$ below ceiling with $\boxed{\omega = \sqrt{81.\dot{6}}}$

(ii) Period $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{81.\dot{6}}}$ seconds

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(iii) Max velocity is at $x = 0$

$v^2 = \omega^2(A^2 - x^2) \Rightarrow \underline{v_{max} = \omega A}$

Need A particle released $l_0 + .08$ below ceiling.

$\Rightarrow x = l_0 + .08 - l_0 - .12 = -.04$ where $v = 0$

$\Rightarrow \boxed{A = .04}$ metres.

$\therefore v_{max} = \sqrt{81.\dot{6}}(.04)$

$= 0.3615$ m/s.